INTERNATIONAL ECONOMIC REVIEW

May 2011
Vol. 52, No. 2

THE TREND IN RETIREMENT*

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A model with leisure production and endogenous retirement is used to explain declining labor force participation rates of elderly males. The model is calibrated to cross-sectional data on labor force participation rates of U.S. males by age, their drop in consumption, and leisure good expenditure share in 2000. Running the calibrated model for the period 1850–2000, a prediction of the evolution of the cross-section is obtained. The model accounts for more than 87% of the increase in retirement of men over 65. The increase in retirement is driven by rising wages and falling prices of leisure goods.

1. INTRODUCTION

Senior citizens in the United States today spend their time gardening, traveling, and enjoying a wide range of entertainment goods. Less than 20% are in the workforce. Instead they allocate their time among various leisure and home activities. The United States was quite a different place in 1880, when more than 75% of men over the age of 65 were participating in the labor market. Labor force participation rates of men aged 65 and over have been continually declining since the latter half of the 19th century. Concurrently, life expectancies have risen, resulting in an increase in the fraction of a man's life spent in retirement.

Retired men spend the majority of their time engaged in leisure activities. Thus, the story of retirement is a story of leisure. Leisure activities, like most activities, require the use of both time and goods. Becker provides examples of such activities in his 1965 paper, one of which is "the seeing of a play, which depends on the input of actors, script, theatre and the playgoer's time."[2] Another example is riding a bike, which requires both time and a bike.

* Manuscript received in June 2008; revised in September 2009.

[1] I owe many thanks to Jeremy Greenwood. I would also like to thank the faculty and graduate students at the University of Rochester, especially Mark Aguiar, Mark Bils, Fatih Guvenen, Guillaume Vandenbroucke, and those who participated in the macro workshop and Jeremy's first and second year macro classes; three anonymous referees; the editor; conference participants of the 2005 SITE "Nexus between Household Economics and the Macro Economy" workshop, the 2006 Labor Supply and Productivity Life Cycle Workshop at the Bank of Canada, the 2006 Midwest Macro Meetings, the 2006 SED meetings; and seminar participants at Concordia University, the Federal Reserve Banks of St. Louis, Dallas, Richmond, and Cleveland, the University of Guelph, York University, the University of Texas at San Antonio, Universidad Carlos III de Madrid, Cornell University, the University of Western Ontario, UCSB, Purdue University, the University of Illinois at Urbana–Champaign, and Stanford University. All errors and inconsistencies are my own. Please address correspondence to: Karen A. Kopecky, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree St. NE, Atlanta, GA 30309-4470. Phone: +404 498 8974. Fax: +404 498 8956. E-mail: *karen.kopecky@atl.frb.org*.

[2] Becker (1965, p. 495).

The quality-adjusted relative price of leisure goods has been declining since the 19th century. Over the same time period, real wage rates have been rising. The argument put forth here is that, together, declining leisure good prices and rising wages have made the leisure-intensive retirement lifestyle more affordable, driving a rise in retirement. The objective of the article is to quantify the contributions of the rise in real wages and the fall in the prices of leisure goods to the decrease in the labor force participation rates of elderly U.S. males throughout the 20th century.

In order to achieve this goal, a continuous-time model in which agents choose the moment of their retirement is developed. In the spirit of Becker (1965), agents in the model economy produce leisure by combining leisure time with leisure goods. When working, agents are assumed to work full-time and allocate remaining time to leisure production. Once retired, agents allocate all their time to leisure production. Agents require a minimum level of market consumption for survival and derive utility from a nonseparable function of leisure and consumption of market goods beyond subsistence level. The model is designed to be consistent with three important characteristics of the retirement period: (i) upon retiring men significantly increase their time spent on leisure activities; (ii) for the majority of workers, retirement is a complete withdrawal from the labor force; and (iii) for many individuals market consumption changes discretely at the moment of retirement.

The model permits leisure goods and leisure time to be either (Hicksian) substitutes or complements in leisure production. It is shown that when the two inputs are complements, a fall in the price of leisure goods relative to leisure time will generate an increased demand for leisure time, lowering the optimal retirement age. In addition, it is shown that when leisure goods and leisure time are complements, the income effect of an increase in real wages dominates the substitution effect. Consequently, higher wages also lower the optimal retirement age. On the other hand, under substitutability between leisure time and leisure goods, a decrease in leisure good prices generates a rise in the optimal retirement age, whereas an increase in wages has an ambiguous effect. The degree of complementarity between leisure goods and leisure time will not be assumed but determined by the calibration. Hence the calibration will determine both the overall and the relative importance of falling leisure good prices and rising wages for the increase in retirement observed in the data.

The model economy consists of overlapping generations of agents. In order to generate variation in the age of retirement within a generation, agents differ by education type, and within education types they vary by initial market productivity level. Each agent has a hump-shaped market productivity profile that depends upon his birth year, education type, and initial market productivity level and an age-specific survival function that depends upon his birth year. In addition, agents vary in their ability to produce leisure or leisure productivity, which is constant over their lifetime and uncorrelated with their market productivity profile. Agents with higher education levels on average have higher levels of market productivity and profiles that peak later in life.

The effect of an increase in an agent's overall level of market productivity is equivalent to the effect of an increase in wages. Therefore, everything else identical, agents with higher overall levels of market productivity will choose to retire earlier than those with lower ones whenever the income effect of an increase in wages dominates the substitution effect. The later peak in the higher types' profiles, however, will increase the marginal cost of retiring at a given age relative to the cost for an agent with an earlier peaking profile. This is because the level of earnings that the agent forgoes to retire is higher. Consequently, although variation in education types and productivity levels within types will generate variation in retirement ages, the relationship between education type, initial market productivity, and retirement age will depend on the calibration.

Everything else identical, agents with higher leisure productivity will retire later than those with lower leisure productivity when leisure time and leisure goods are complements, and vice versa when they are substitutes. When leisure time and leisure goods are complements, agents with higher leisure productivity demand more leisure goods at each moment of their life. Thus it is optimal for them to delay retirement in order to increase their lifetime earnings and, consequently, their expenditure on leisure goods. When leisure time and leisure goods are substitutes, it is the lower productivity agents that have a higher demand for leisure goods and therefore choose to retire relatively later.

The model is calibrated to the year 2000 using cross-sectional data from the Health and Retirement Study (HRS). The calibration is done by minimizing the distance between the model and data along eight key moments: the labor force participation rates of six age groups, the median drop in consumption at retirement, and leisure goods' expenditure share. Then the model is used to compute the labor force participation rates of the six age groups over the 1850–2000 period by plugging in the rate of decrease of leisure good prices and the rate of increase of wages along with the changes in agents' survival profiles, life expectancies, and education levels over this period.

The model is able to match the year 2000 distribution of elderly labor force participation rates by age group and generates a consumption drop at retirement and leisure good expenditure share in 2000 that are in line with the data. Under the baseline calibration, leisure time and leisure goods are complements, and thus the fall in the relative price of leisure goods and the rise in wages over the 1850–2000 period have a positive impact on retirement. An increase in the fraction of agents with high school and college educations also positively impacts retirement. However, the effect of rising education is small. Finally, under the assumption that agents can fully insure against survival risk, rising life expectancies in the model have a negative impact on retirement.

According to the model, taking into account the observed changes in survival profiles, life expectancies, and education levels since the 18th century, the rising wage rate and falling prices of leisure goods explain more than 87% of the rise in retirement of males aged 65 and over. The model also reveals that these driving forces had a large impact on the retirement behavior of men aged 55–64. A series of counterfactual experiments show that the rise in real wages was the dominant force decreasing labor force participation rates. However, the decrease in the price of leisure goods since the 18th century has also played a significant role, alone generating approximately 13% of the increase in retirement of the elderly ages 65–69.

This article is a first attempt at accounting for the long-run rise in retirement using a quantitative macroeconomic approach. Similar arguments on the impact of leisure goods' prices on labor supply have been made to understand changes in labor supply on the intensive margin by Owen (1971) and more recently by Vandenbroucke (2009) and González Chapela (2007). The argument that a fall in the price of leisure goods may be an important driver of the long-run rise in retirement was first made by Costa (1998). The most common alternative theory of rising retirement is that it was driven by the increase in social insurance programs and private pensions. However, findings from empirical studies on the ability of such programs to account for the rise are mixed.

The article proceeds as follows. Section 2 presents some facts on retirement and leisure. Section 3 presents the model. The quantitative experiment is presented in Section 4, which includes of explanation of the calibration procedure and presents the model's prediction for the trend in retirement since 1850. The section concludes with the presentation of a series of counterfactual experiments and a discussion of the contribution of the various driving forces to the retirement trend. Section 5 discusses related literature, and Section 6 concludes.

2. RETIREMENT

Retirement is defined as a planned, complete, and usually permanent withdrawal from the labor force by older workers. In this section, data illustrating the trends in retirement in the United States and other countries is presented. The section then provides a discussion of some important characteristics of the retirement period that are used as guidelines for making modeling assumptions.

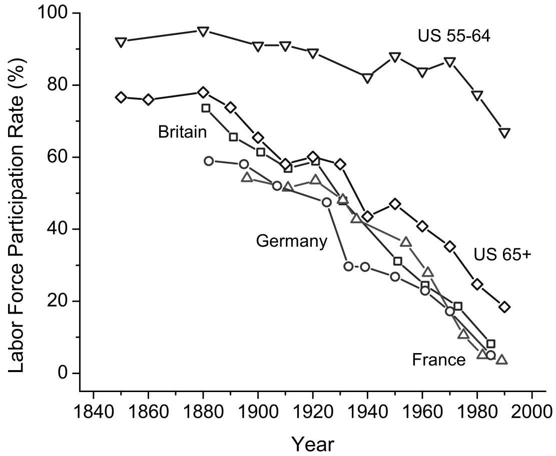

FIGURE 1

LABOR FORCE PARTICIPATION RATES OF MEN AGED 65 AND OVER FOR THE PERIOD 1850–1990 IN THE UNITED STATES, FRANCE, GREAT BRITAIN, AND GERMANY AND MEN AGED 55–64 IN THE UNITED STATES

2.1. *Historical Trends.* A trend of rising retirement since the 19th century is not unique to the United States. Figure 1 shows the labor force participation rates of men aged 65 and over for the period 1850–1990 in the United States, France, Great Britain, and Germany, and the participation rates of men aged 55–64 in the United States. Notice that the decline in the labor force participation rates occurred in all four countries. This decline cannot be accounted for by the change in the composition of the elderly population due to the increase in life expectancy. Participation rates fell for all ages above 65. In addition, participation rates have fallen among men aged 55–64. In 1880, 96% of men aged 60–64 were in the labor force; by 1990 only 39% were. For men in their late 50s, participation rates have been declining since 1900 but started to decline at a faster rate around 1960.[3]

Labor force participation rates have also been declining in developing countries. For example, the labor force participation rate of men aged 65 and over fell from 67% to 52% in Mexico between 1970 and 1999. In Peru it fell from 62% to 41% and in Turkey from 68% to 34% over the same period. Unfortunately, data from these countries is only available for recent years.[4]

In order to obtain a direct measure of the increase in retirement, a statistic called the retirement rate is calculated using data from IPUMS for men aged 50 and over for the period 1850–2000. The retirement rate is the ratio of the number of men who are retired to the number of men either in the labor force or retired. In order to be classed as retired a man must be completely out of the labor force. Hence, men who are working part-time or part-year are counted as working and not retired. The retirement rates are presented in the left-hand graph of Figure 2 for men by five-year-age groups.[5] Notice that the retirement rates of the youngest age group, those aged 50–54, do not increase over time whereas the rates of all the other age groups do increase. For the oldest age group, those aged 75–79, the retirement rate rises from about 20% in 1850 to nearly 90% in 2000.

The combination of rising life expectancies and declining labor force participation rates of the elderly has led to an increase in the expected duration of retirement. In fact, a 20-year-old

[3] See Costa (1998, chapter 2) for a in-depth discussion of trends in labor force participation. The source for Figure 1 is Costa (1998, p. 29, Tables 2A.1 and 2A.2).

[4] *Source:* U.S. Census Bureau, Series P95/01-1, *An Aging World: 2001* (2001).

[5] The data for the retirement rates is from: Ruggles, et al. 2004. *Integrated Public Use Microdata Series: Version 3.0.* (IPUMS) Minneapolis, MN: Minnesota Population Center. It can be found at http://www.ipums.org. The retirement rates for each age group were computed by observing that: % retired = (% not in the labor force − % never participating)/(1 − % never participating).

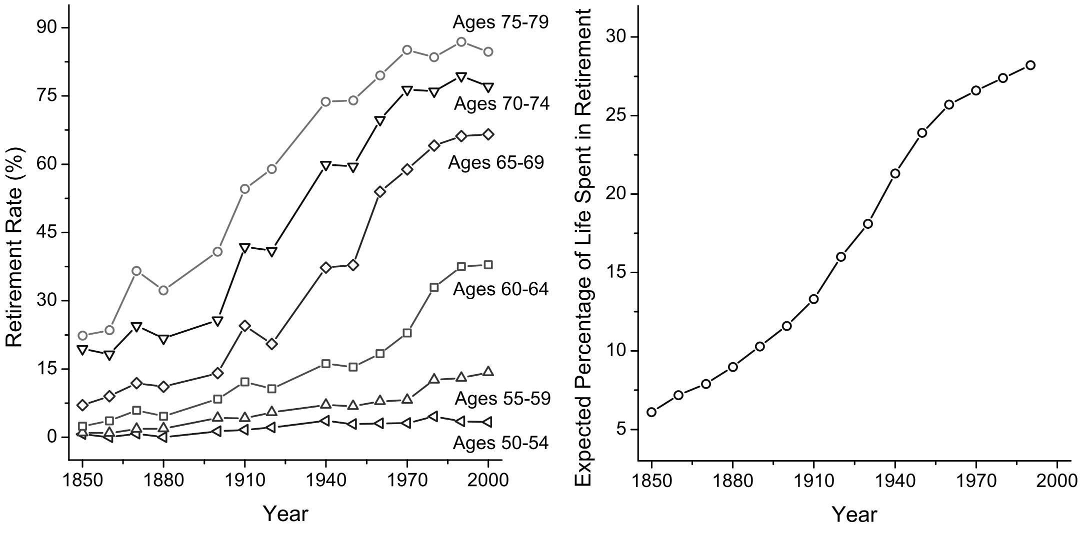

FIGURE 2

RETIREMENT RATES FOR MEN AGED 50 AND OVER BY AGE GROUP AND THE EXPECTED PERCENTAGE OF LIFE SPENT IN RETIREMENT AT THE AGE OF 20 FOR THE PERIOD 1850–2000 IN THE UNITED STATES

TABLE 1
HOURS PER WEEK SPENT IN VARIOUS ACTIVITIES FOR MEN BY AGE GROUP IN 1985

	Age		
Activity	25–54	55–64	65+
Sleeping	54.9	57.5	58.7
Working or commuting	40.1	23.7	8.0
Recreation	35.8	42.7	51.1
Grooming and child care	10.9	10.2	12.3
Eating and preparing meals	9.5	12	12.6
House and yard work	9.2	13.5	16.7
Shopping	4.7	5.4	5.6
Other	2.1	2	2.4

male in 1850 would have expected to spend approximately 6% of his adult life retired, whereas a male who was 20 in 1990 can expect to spend 30% of adult life retired. The right-hand graph in Figure 2 shows how the expected percentage of adult life spent in retirement has risen over this period.[6]

2.2. *Characteristics of the Retirement Period.* In order to study the impact of changing prices on the retirement behavior of men, a model of retirement must be consistent with the defining characteristics of the retirement period. Three important characteristics are discussed below along with an explanation of how the baseline model is designed to be in accordance with them.

2.2.1. *Increase in leisure.* The retirement period is a period in which one must reallocate his time from market to nonmarket activities. Thus, to gain insight into the retirement decision it is important to investigate how retired people spend their time. Table 1 gives a breakdown of men's time use by age.[7] Notice that older men allocate more of their time to leisure and home

[6] Adult life excludes the first 20 years. The data for the expected portion of life in retirement in Figure 2 is taken from Lee (2001, p. 645, Table 1). It is based on the same IPUMS data as used to compute the retirement rates. The expected length of retirement is computed assuming 20-year-olds have perfect information about future mortality rates.
[7] Source for Table 1 is Godbey and Robinson (1997, p. 207, Table 19).

TABLE 2
HOURS PER WEEK SPENT IN VARIOUS LEISURE ACTIVITIES FOR MEN BY AGE GROUP IN 1985

	Age		
Activity	25–54	55–64	65+
Participating in organizations	0.9	2.2	1.1
Attending events	0.9	0.6	0.1
Visiting	6.6	6.7	6.0
Playing or watching sports	2.9	2.8	3.0
Hobbies	2.5	3.5	3.5
Talking or socializing	2.8	3.1	4.5
Watching TV	16.1	18.2	24.9
Reading	2.6	4.7	6.7
Listening to music	0.5	0.9	1.3

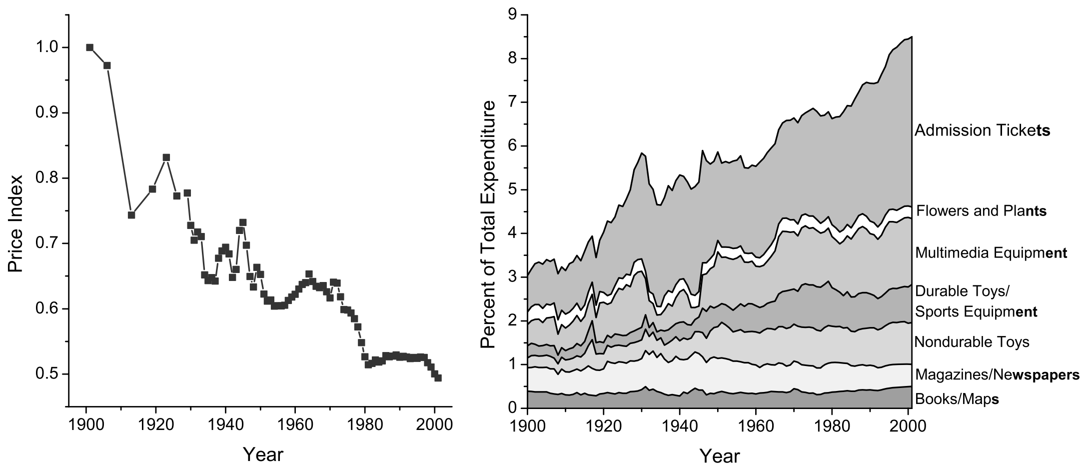

FIGURE 3

RELATIVE PRICE INDEX OF LEISURE GOODS AND BREAKDOWN OF THEIR SHARE OF TOTAL EXPENDITURE IN THE UNITED STATES FOR THE PERIOD 1900–2001

activities. In particular, men age 55–64 spend approximately 19% more time on recreation than men aged 25–54, whereas men age 65 and over spend nearly 43% more time. Thus retirement is a period when men's time spent on leisure activities significantly increases. Consistent with this fact, retired men spend more time using leisure goods, such as televisions, radios, stereos, books, magazines, and newspapers. For example, according to Godbey and Robinson (1997), in 1985, men aged 55–64 spent 13% more time watching television than men aged 25–54, whereas men over the age of 64 spent 81% more time. Men over the age of 64 also spent nearly double the amount of time men aged 25–54 spent reading and listening to music. Table 2 gives a breakdown of time spent in various leisure activities by age groups for men in 1985.[8] In addition to spending more time with leisure goods, there is evidence that upon retirement, individuals increase the share of their expenditure that they allocate to leisure goods. Weagley and Huh (2004) find, using data from the 1995 Consumer Expenditure Survey, that controlling for age, education, income, and demographics, leisure goods' share of total expenditure increases at retirement.

How have leisure good prices changed over time? The left-hand side (LHS) of Figure 3 presents the price index for a particular selection of leisure goods relative to the CPI over the period 1900–2000. The price of these leisure goods has fallen at an average annual rate of

[8] The source for Table 2 is the same as that for Table 1. See footnote 7.

approximately 1%.[9] The price index is based on the set of leisure goods whose expenditure shares are provided in the graph on the right-hand side (RHS).[10] In 1900 the average American allocated approximately 3% of his expenditure to leisure goods. By 2001 this fraction had increased to over 8%. Notice that this set of leisure good does not include transportation goods or services. Yet approximately 30% of the average total miles driven with a car each year are driven for social and recreational trips.[11] When 30% of expenditure on transportation is included in leisure good expenditure, leisure goods' expenditure share rises from about 4% in 1900 to nearly 12% in 2001.

In order to capture the effect of men's reallocation of time from market activities to leisure activities upon retirement, it is assumed that men engage in leisure production. The notion of leisure production is inspired by Becker's 1965 paper and is similar to home production. There is an extensive literature demonstrating the importance of home production in explaining a variety of phenomena.[12] The key difference between home and leisure production is the following. Time spent on housework and household durables are usually found to be substitutes in production of the home good. Thus, a fall in the price of household durables decreases the demand for time spent on housework. In contrast, time spent on leisure activities and leisure goods are argued here to be complements in the production of leisure. Under this assumption, a fall in the price of leisure goods leads to an increased demand for leisure time.

2.2.2. *Labor force withdrawal.* For the majority of workers, retirement involves a complete withdrawal from the labor force or, in other words, switching from full-time work to being fully retired. A variety of theories have been proposed to explain why a majority of older workers withdraw once and completely from the labor market. They include the inability of older workers, in demanding jobs, to handle physical and/or mental stress, minimum hours constraints, and schedule inflexibility, and employer incentives and pensions. Evidence from the HRS has pointed to minimum hours constraints and schedule inflexibility as the largest factors influencing retirement decisions. For example, Hurd and McGarry (1993) find, using the HRS, that the ability to change hours of work, pensions, and health insurance have an important effect on retirement decisions, whereas Gustman and Steinmeier (2004) conclude, based on the HRS, that relaxing minimum hours constraints would significantly increase the percentage of older people who continue working. Given these findings, it is assumed that agents in the model start off their lives as workers and are unable to adjust labor supply on the intensive margin. Consequently, agents in the model are either working full-time or retired.

2.2.3. *Drop in market consumption.* Numerous studies based on a variety of different data sets have found evidence of a significant drop in consumption at the moment of retirement. The causes of this drop are not well understood, and thus it has come to be known as the retirement-consumption puzzle. In contrast to these studies, in a recent work, Hurd and Rohwedder (2008) find an average consumption drop for individuals of 4.7% of preretirement expenditure and a median drop of 5.9%. Their estimates are based on data from the HRS and three waves of a supplemental survey called CAMS. Their analysis is unique in that it is the first study of the consumption drop at retirement for U.S. households that is based on observations of total expenditure before and after retirement by the same individuals. The findings are in contrast to those of earlier works that used synthetic panels and/or partial measures of consumption

[9] Sources for the price index: For the period 1901–34, data from Owen (1969, p. 85, Table 4-B) are used; for the period 1935–68, data are from the Historical Statistics Series E 165; and for 1969–2001, the data are taken from the Bureau of Labor Statistics' *Handbooks of U.S. Labor Statistics*, second edition (1998, p. 263) and sixth edition (2003, p. 308).

[10] The source for the expenditure shares is Lebergott (1996).

[11] Based on data from a selection of years between 1951 and 1995. Source: 1951–58: U.S. Department of Commerce, Bureau of Public Roads. 1969–95: Federal Highway Administration, *National Personal Transportation Survey, Summary of Travel Trends*.

[12] For examples see Reid (1934), Benhabib et al. (1991), Greenwood et al. (2005), Ríos-Rull (1993), and the references therein.

and estimated the average drop in market expenditure to be in the range of 10–30% of pre-retirement expenditure.[13] Exploring the distribution of the expenditure drop across individuals, Hurd and Rohwedder (2008) find that estimates based on synthetic cohorts are likely to have been driven by a few individuals having large declines. They also find a great deal of variation in expenditure changes at retirement across the population, with some individuals experiencing significant increases and others significant decreases.

One possible explanation for the discrete changes in expenditure at retirement observed in the data is that they are driven by complementarity or substitutability in utility between nonmarket time and various market goods. Supporting this view, Aguiar and Hurst (2005) find that whereas expenditure on food declines at retirement, calorie and vitamin consumption does not, suggesting that retirees substitute nonmarket time for expenditure on food. The findings of Weagley and Huh (2004) that leisure goods expenditures increase at retirement support the notion that leisure goods and nonmarket time are complements in utility.

Complementarity (or substitutability) in utility, also known as Edgeworth-Pareto complementarity, is different from Hicksian complementarity. Two items are complements (substitutes) in utility if an increase in the amount of one item increases (decreases) the marginal utility of the other, whereas, two items are Hicksian complements (substitutes) if a decrease in the price of one increases (decreases) demand for the other. For two items to be Edgeworth-Pareto complements or substitutes they must be nonseparable in utility such that their marginal utilities are functions of the level of the other item. When this is the case, it is optimal to discretely adjust the level of one item in response to a discrete jump in the level of the other in order to smooth utility. Whether two goods are complements or substitutes in utility depends upon the elasticity of substitution between the two items and the concavity of the utility function.

In the model economy, total expenditure consists of expenditure on leisure goods and a general consumption good. Agents are assumed to produce leisure by combining leisure time with leisure goods, and the production function is such that leisure time and leisure goods can be either Hicksian substitutes or complements with one another. In this framework, the marginal utility of leisure time will depend on the quantity of leisure goods and vice versa. When agents retire, their leisure time jumps up, generating a discrete jump in their leisure good consumption. If utility is separable in market consumption and leisure, then the jump in leisure good consumption will drive the change in total consumption at the moment of retirement. However, as mentioned above, leisure good expenditure increases at retirement. In order to give the model the chance to be consistent with this fact and match the drop in total expenditure observed in the data, market consumption and leisure are assumed to be nonseparable in utility.

3. THE MODEL

Time is continuous and indexed by t. The economy consists of overlapping generations. Agents are characterized by their type $s \equiv (\tau, e, x_0, z)$, where τ denotes the date of the agent's birth. An agent born at moment τ will be age $a = t - \tau$ at time t. The parameter e denotes the agent's level of education, and x_0 is the agent's initial market productivity level. Together τ, e, and x_0 determine the agent's lifetime market productivity profile $x_s(\cdot)$. The parameter z is the agent's ability to produce leisure and is constant over the agent's lifetime.

Within each generation there is a distribution of agents across education levels, and for each education level there is a distribution of agents across initial market productivity levels. Let $F_\tau(e)$ denote the distribution of generation-τ agents across education levels and $G_e(x_0)$ denote

[13] For example, Hurd and Rohwedder (2005) find, using data from the 2001 CAMS supplemental survey of 2000 HRS respondents, that expenditure on market nondurables drops by 16.8% for singles and 11.6% for married couples at the moment of retirement of the household head, Bernheim et al. (2001) find an average drop in expenditure on food of 14% using data from the Panel Study of Income Dynamics, and Aguiar and Hurst (2005) find an average drop in expenditures on food of 17% using data from the Continuing Survey of Food Intakes conducted by the U.S. Department of Agriculture.

the distribution of agents of education level e across initial productivity levels. Each agent's leisure productivity, z, is drawn from the distribution $H(z)$, which is independent of the agent's age cohort, education level, and initial market productivity. In addition, there is no correlation between an agent's market ability profile and his leisure ability.

Agents live for a maximum length of time T. At ages below T an agent's survival is determined by a generation-specific survival function $q_\tau(a)$. In other words, $q_\tau(a)$ is the probability that a member of the generation-τ cohort survives to at least age a.

The economy contains two types of goods: market (or general consumption) goods and goods that aid in leisure production, here called leisure goods. The price of market goods at each date t is normalized to one. The price of leisure goods relative to market goods at date t is denoted by $p_g(t)$.

3.1. *Agents' Maximization Problem.* Agents have one unit of time at each moment of their lives. Newly born agents of type s start off their lives as workers, inelastically supplying a fraction $\bar{h}$ of their time to the market and receiving earnings $w(\tau + a)\bar{h}x_s(a)$. The function $w(\tau + a)$ is the wage per an efficiency unit of labor at time $\tau + a$, and $x_s(a)$ is the agents' market productivity at age a. Market productivity profiles are humped-shaped over the life cycle.

Time that is not spent working on the market is dedicated to the production of leisure, which requires both time and leisure goods. At each age a, each agent combines leisure with market goods to generate utility. In addition to choosing the stream of market goods, $c_s(a)$, and the stream of leisure goods, $g_s(a)$, that he purchases over his lifetime, a type-s agent chooses an age at which to permanently retire from market work, A_s. Once retired, agents spend all their time on leisure production. Thus, time spent on leisure production by a type-s agent is defined as

$$l_s(a) = \begin{cases} 1 - \bar{h}, & a \leq A_s, \\ 1, & a > A_s. \end{cases} \tag{1}$$

Leisure time and leisure goods are combined to produce leisure using the constant elasticity of substitution production function

$$n_s(a) = \left\{\zeta g_s(a)^\chi + (1 - \zeta)[z l_s(a)]^\chi\right\}^{\frac{1}{\chi}},$$

where $0 \leq \zeta \leq 1$, $\chi \leq 1$, and $\chi = 0$ implies a Cobb–Douglas production function. The parameter ζ is the weight on leisure inputs relative to leisure time in the production function. Under this formulation, the elasticity of substitution between leisure time and leisure goods is $1/(1 - \chi)$.

An agent born at date τ with education level e, initial market productivity x_0, and leisure productivity z chooses paths of market good and leisure good purchases over his lifetime, $c_s(a)$ and $g_s(a)$, respectively, and the age of his retirement, A_s, to maximize his expected lifetime utility given by

$$\int_0^{A_s} e^{-\theta a} q_\tau(a) U[c_s(a), n_s(a)]\, da + \int_{A_s}^{T} e^{-\theta a} q_\tau(a) U[c_s(a), n_s(a)]\, da, \tag{2}$$

subject to his lifetime budget constraint and $A \leq T$. The parameter θ captures the subjective time-discounting rate, and the cohort-dependent survival function, $q_\tau(a)$ is log-sextic in age. The momentary utility function is of the constant relative risk aversion form, so

$$U[c_s(a), n_s(a)] = \frac{\left\{[c_s(a) - \hat{c}]^\alpha\, n_s(a)^{1-\alpha}\right\}^{1-\sigma}}{1 - \sigma}, \tag{3}$$

where $\hat{c} \geq 0, 0 \leq \alpha \leq 1, \sigma > 0$, and $\sigma = 1$ implies log-utility. The parameter α determines the importance of market goods relative to leisure for utility, and the parameter $\hat{c}$ is a subsistence level of market good consumption. The objective function is expressed as the sum of the agent's utility while working and his utility while retired. This is done to highlight the role that the age at which the agent retires, a choice variable, plays in his decision problem. It is also written in this way because, as is described below, time spent on leisure is not continuous at the moment of retirement. In general, the discontinuity in leisure time at retirement will result in a discontinuity in the momentary utility function at retirement.

As in Kalemli-Ozcan et al. (2000), the economy contains a life-insurance company that offers actuarially fair annuities to the agents. Annuities allow agents to share mortality risk, and, as was first shown by Yaari (1965), since the agents have no bequest motive or precautionary savings motive, they will use annuities as their sole instrument of investment. The rate of return on an annuity for an agent with death hazard rate $h_\tau(a) = -d \ln q_\tau(a)/da$ is equal to r, the risk-free rate, plus $h_\tau(a)$. Thus the agent's lifetime budget constraint is

$$\begin{aligned}(4)\qquad &\int_0^{A_s} e^{-ra} q_\tau(a) c_s(a)\, da + \int_{A_s}^{T} e^{-ra} q_\tau(a) c_s(a)\, da + \int_0^{A_s} e^{-ra} q_\tau(a) p_g(a+\tau) g_s(a)\, da \\ &\quad + \int_{A_s}^{T} e^{-ra} q_\tau(a) p_g(a+\tau) g_s(a)\, da = \int_0^{A_s} e^{-ra} q_\tau(a) x_s(a) \bar{h} w(a+\tau)\, da.\end{aligned}$$

Hereafter the s-subscript is dropped for ease of notations.

The first-order condition for market consumption is

$$(5)\qquad \alpha e^{-\theta a}[c(a) - \hat{c}]^{(1-\sigma)\alpha - 1} n(a)^{(1-\sigma)(1-\alpha)} = \lambda e^{-ra}, \quad \forall a \in [0, T],$$

where λ is the multiplier on (4) in the Lagrangian. The first-order condition for purchase of the leisure good is

$$\begin{aligned}(6)\qquad &(1-\alpha)\zeta e^{-\theta a}[c(a) - \hat{c}]^{(1-\sigma)\alpha} n(a)^{(1-\sigma)(1-\alpha)-\chi} g(a)^{\chi - 1} \\ &\quad = \lambda e^{-ra} p_g(a+\tau), \quad \forall a \in [0, T].\end{aligned}$$

Notice that time spent on leisure enters the first-order conditions for market consumption and leisure goods. This occurs because market consumption and leisure time are nonseparable in utility, as are leisure goods and leisure time. Hence, leisure time affects the marginal utility of market consumption and leisure goods. Agents want to smooth marginal utility over their lifetime. However, at the moment of retirement their marginal utility jumps discretely due to the discrete jump in their leisure time. In order to smooth marginal utility, the agents make discrete adjustments to their consumption and leisure goods at this moment. The first-order condition for the retirement age is

$$\begin{aligned}(7)\qquad \frac{\{[\underline{c}_A - \hat{c}]^\alpha \underline{n}_A^{1-\alpha}\}^{1-\sigma}}{1-\sigma} - \frac{\{[\bar{c}_A - \hat{c}]^\alpha \bar{n}_A^{1-\alpha}\}^{1-\sigma}}{1-\sigma} \le \lambda e^{(\theta - r)A} \big[&x(A)\bar{h} w(A+\tau) - (\bar{c}_A - \underline{c}_A) \\ &- p_g(A+\tau)(\bar{g}_A - \underline{g}_A)\big],\end{aligned}$$

where $\bar{c}_A$ is market consumption at the moment of retirement, given that the agent is still working, or

$$(8)\qquad \bar{c}_A = c(A),$$

$\underline{c}_A$ is defined as

$$(9)\qquad \underline{c}_A = \lim_{a \to A^+} c(a),$$

and $\overline{n}_A$, $\underline{n}_A$, $\overline{g}_A$, and $\underline{g}_A$, are defined similarly. In order to understand Equation (7), consider the problem of an agent who is deciding whether or not he should retire at age A. If he retires, his instantaneous utility changes. His net gain in utility is on the LHS of Equation (7). This is the marginal cost of postponing retirement. The RHS is the marginal benefit. It is the utility value of the savings of the agent at age A if he is working net of his savings at A if he is retired. As long as the marginal benefit of working exceeds the marginal cost, the agent will not retire. Thus an agent could die having never retired. At an interior solution for the optimal retirement date, A, Equation (7) will hold with equality.

Solving (5) for $c(a)$ and differentiating with respect to a gives

$$\frac{\dot{c}(a)}{c(a)-\hat{c}} = \frac{1}{\Phi}\left[\theta - r - \Psi\frac{\dot{n}(a)}{n(a)}\right], \tag{10}$$

where

$$\Phi = (1-\sigma)\alpha - 1,$$

and

$$\Psi = (1-\sigma)(1-\alpha).$$

Totally differentiating (6) with respect to g and a, plugging in (10), and rearranging yields

$$\frac{\dot{g}(a)}{g(a)} = \left(\frac{\dot{p}_g(a+\tau)}{p_g(a+\tau)} - \frac{\theta - r}{\Phi}\right)\Big/\left(\chi - 1 - \zeta\left(\frac{\Psi}{\Phi}+\chi\right)\left[\frac{g(a)}{n(a)}\right]^{\chi}\right). \tag{11}$$

The stream of market goods and leisure goods that agents purchase over the lifetime must satisfy the differential equations given by Equations (10) and (11).

Under what conditions will falling prices of leisure goods and rising wages generate an increase in retirement? First consider the effect of a fall in the price of leisure goods. The lower price will lead to an increase in retirement when $\chi < 0$. In this case leisure time and leisure goods will be complements and a decrease in the price of leisure goods will generate an increased demand for leisure time. Leisure time, however, can only be increased by retiring earlier. Thus agents will choose to exit the labor force at a younger age. In the opposite case, $\chi > 0$, leisure goods and leisure time are substitutes and a decrease in the price of leisure goods will delay retirement.

Now consider the impact on retirement of an increase in wages. The impact can be decomposed into two effects. The first effect is due to the fact that, when the wage rate increases, the price of leisure goods relative to leisure time falls. This has the same effect on retirement as a fall in the price of leisure goods: If $\chi < 0$, then it will lower the retirement age and if $\chi > 0$, it will raise the retirement age. The second effect is due to the fact that the price of market consumption relative to leisure time falls. As long as $\hat{c} > 0$, market goods and leisure time will be complements, and the fall in this price will lower the retirement age. If $\hat{c} = 0$, then the retirement age is independent of the relative price. Hence, overall, an increase in wages will lead to an increase in retirement if $\chi \leq 0$. However, when $\chi > 0$, the two effects work in opposite directions, and, depending on which effect dominates, retirement will either decrease or increase. These results are formalized in the following two propositions. Formal proofs are provided in the Appendix for the case where $\sigma = 1$. Although not proven here, the results hold for the general case of $\sigma > 0$. In the numerical exercise that follows, whether χ is positive or negative will be determined by the calibration.

PROPOSITION 1. *Let the path of leisure good prices over the agent's lifetime be* $p_g(a+\tau) = p_{g,\tau}\tilde{p}_g(a+\tau)$*. Then, at an interior solution, the retirement age A is*

(i) *independent of* $p_{g,\tau}$ *if* $\chi = 0$,
(ii) *increasing in* $p_{g,\tau}$ *if leisure goods and leisure time are complements* ($\chi < 0$),
(iii) *decreasing in* $p_{g,\tau}$ *if leisure goods and leisure time are substitutes* ($\chi > 0$).

PROOF. See the Appendix for the proof when $\sigma = 1$.

PROPOSITION 2. *Let the path of wages over the the agent's lifetime be* $w(a+\tau) = w_\tau\tilde{w}(a+\tau)$*. Then, at an interior solution with* $\chi \leq 0$*, the retirement age A is decreasing in* w_τ.

PROOF. See the Appendix for the proof when $\sigma = 1$.

Variation in market productivity profiles and leisure productivity will generate a variation in retirement rates within generations. An agent's retirement age will depend upon both the level and the shape of his productivity profile. Notice that the effect on retirement age of a higher level of market productivity, holding the shape of the profile fixed, is equivalent to the effect of an upward shift in an agent's lifetime wage profile. Thus, holding the shape fixed, agents with higher levels of market productivity will retire later when χ is negative; however the impact is ambiguous when χ is positive. This result is formalized in the following corollary to Proposition 2.

COROLLARY 1. *All else being the same, at an interior solution with* $\chi \leq 0$*, an upward level shift in an agent's productivity profile will lead to a decrease in his retirement age A.*

PROOF. Assume that w_τ in Proposition 2 is the shift in the agent's productivity profile instead of in his wage profile. ■

In the numerical experiment that follows, higher market productivity agents will have productivity profiles that peak later in their lifetime and reach a higher overall level relative to their initial productivity. A later peaking profile will increase the optimal age of retirement. This is because, initial productivity being the same, the forgone income and thus the marginal cost of retiring at an age later in life for an agent with a later peaking profile is higher. Whether a higher productivity type retires earlier or later than a lower type will depend on whether the effect of the level or the shape of the profile dominates.

Compared to market productivity, a higher level of leisure productivity has the opposite effect on retirement when χ is negative. In this case, complementarity between leisure goods and leisure time increases the lifetime demand for leisure goods. Thus it is optimal for agents with relatively higher leisure productivity to work for more years to finance a higher level of leisure good expenditure. On the other hand, when $\chi > 0$, leisure time is a substitute for leisure goods, and it is the agents with relatively less leisure productivity who have a higher lifetime demand for leisure goods and, consequently, choose to retire later. This result is formalized in the following proposition.

PROPOSITION 3. *At an interior solution the retirement age A is*

(i) *independent of z if* $\chi = 0$,
(ii) *increasing in z if leisure goods and leisure time are complements* ($\chi < 0$),
(iii) *decreasing in z if leisure goods and leisure time are substitutes* ($\chi > 0$).

PROOF. See the Appendix for the proof when $\sigma = 1$.

For the numerical analysis, it is assumed that wages grow at the constant rate κ over time such that $w(t) = w(0)e^{\kappa t}$, and the price of the leisure goods is assumed to decrease over time at rate γ such that $p_g(t) = p_g(0)e^{-\gamma t}$. Since the models purpose is to generate the long-run trend in retirement, these assumptions seem reasonable and greatly simplify the numerical analysis.

4. NUMERICAL ANALYSIS

The following experiment is devised to bring the model to the data. First, the model is calibrated to the year 2000 by matching data on the cross-sectional distribution of labor force participation rates, the drop in market consumption at the moment of retirement, and the average expenditure share on leisure goods in 2000. Then the rates of change of wages and leisure good prices, cohort-specific survival functions, productivity profiles, and distributions over education levels, also chosen to be consistent with the data, are plugged into the model. Finally, the calibrated model is used to reproduce the evolution of the cross section of the elderly population, aged 50–80, for the period 1850–2000.

4.1. *Computation.* In order to ease the computation of the statistics of interest, the model and statistics are computed for a discrete set of types. Each agent is characterized by his birth year, τ, his education level, e, his initial market productivity, x_0, and his leisure productivity, z. In order to discretize τ, it is assumed that agents are born at 5-year intervals. Education level is discretized by assuming each agent belongs to one of three possible groups: G, H, and C—corresponding to grammar school (8 years of education or less), high school (9–12 years of education), and college (13 years of education or more), respectively.

The set of discrete values for x_0 is determined as follows. First, assume that initial productivity level conditional on being in education group e, x_0^e, can take 1 of 10 possible values. Let $\mathcal{X}^e$ denote the set of such values. Second, assume that for each education group e, the distribution of x_0^e approximates a truncated lognormal distribution. The truncation points are set so that 0.5% of the area underlying the original distribution is removed from each side. Thus $\ln(x_0^e)$ is distributed truncated normal with mean $\mu_{x_0}^e$ and standard deviation $\sigma_{x_0}^e$. Then, assume that, for each education group, $\mathcal{X}^e$ is an evenly spaced in logarithms grid over the domain of education group e's distribution. Finally, the set of all possible values of x_0 is given by $\mathcal{X} = \bigcup_{e \in \{G,H,C\}} \mathcal{X}^e$ and the distribution over $\mathcal{X}$ for education group e, $G_e(x_0)$, is such that it approximates the truncated lognormal with corresponding mean and variance over $\mathcal{X}^e$ and places 0 weight on values outside of $\mathcal{X}^e$.

Finally, the discrete set for z, denoted $\mathcal{Z}$, is assumed to consist of 20 values. Similarly to x_0^e, assume that $\ln(z)$ is approximately truncated normal with mean μ_z and standard deviation σ_z. Let $\mathcal{Z}$ be an evenly spaced grid in logarithms set such that 0.5% of the area underlying the original lognormal distribution is removed from each side and let $H(z)$ be the corresponding weights that approximate the lognormal distribution.

Given an agent's type s and the series for prices and wages, the agent's maximization problem is solved numerically by a combination of a grid search over the retirement date, A, and a more efficient gradient-based root-finding algorithm. Care is taken to ensure that a potential solution is the global maximizer by checking the second-order conditions and corners. A more detailed description of the algorithm used to compute the solution to the agent's maximization problem can be found in the Appendix.

4.2. *Calibration.* Since agents are born at 5-year intervals, at any moment in time the population of people aged 50–79 is represented by six cohorts ages 52, 57, and so on up to 77. Specifically, 52-year-olds in the model represent 50–54-year-olds in data and so on. Agents are born as 20-year-old adults. Therefore, time begins in 1793, since the oldest cohort in the economy, i.e., the one who is age 77 in 1850, must be born 57 years earlier.

The wage rate in 1793 is normalized to 1. The baseline calibration then proceeds in two stages. In the first stage, parameters that can be determined from the data without computing the

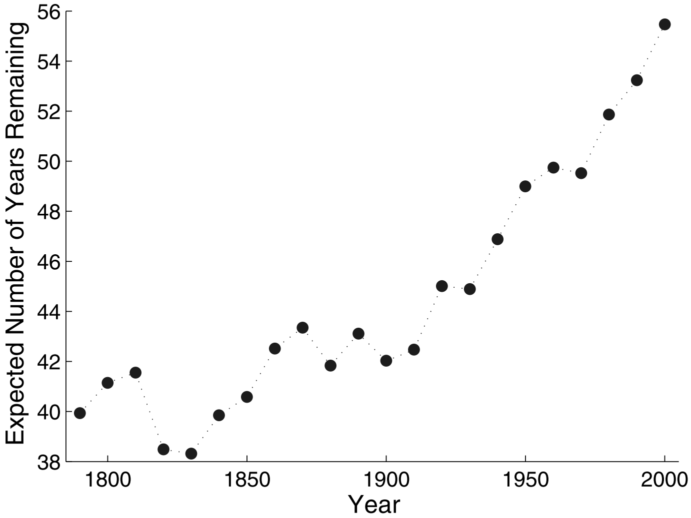

FIGURE 4

EXPECTATIONS OF LIFE FOR 20-YEAR-OLD MALES FOR THE PERIOD 1790–2000

model are assigned. Then in the second stage, termed "estimation," the remaining parameters are chosen to minimize the difference between moments from the model and the data. The "estimation" done here is similar to generalized methods of moments estimation but without optimal weighting or computation of standard errors.

4.2.1. *A priori*

Survival functions and life expectancies. Each cohort's survival function gives the probability of surviving from model age 0 (age 20 in the data) to model age a (age $a + 20$ in the data). The functions take the following functional form:

$$\ln q_\tau(a) = q_\tau^0 + q_\tau^1 a + q_\tau^2 a^2 + q_\tau^3 a^3 + q_\tau^4 a^4 + q_\tau^5 a^5 + q_\tau^6 a^6.$$

The coefficients are estimated from data on survivorship at age 20 for men born after 1850. Age-specific survivorship data is taken from Haines (1994) for the period 1850–1900 and the *National Vital Statistics Report* for the period 1900–2000.[14] Survivorship tables are not available for the 1790–1850 period. Hence, the survival functions are approximated using data on life expectancies based on family histories taken from Pope (1992).[15]

The life expectancies of 20-year-old males that the model is matched to are given in Figure 4. The data shows that the life expectancy of a 20-year-old male in 1790 was similar to that of a 20-year-old male in 1850, and life expectancies fell dramatically for U.S. males (surprisingly also for females) during the antebellum period. In addition, it was not until the 20th century that mortality conditions in the United States began to consistently improve. The data is top-cut at age 100; hence, it is assumed that agents live to a maximum age of 80 in the model, i.e., $T = 80$ for all cohorts. The survival functions of the 1793 cohort, the 1873 cohort, and the 1968 cohort are given in Figure 5. The significant reduction in the probability of death at younger ages for

[14] The exact source for the 1900–2000 period is the "United States Life Tables, 2001" in *National Vital Statistics Reports*, Vol. 52, No. 14 (2004).

[15] Specifically, for each decade from 1790 to 1840 the survival function is determined by shifting the 1850 survival function to match the decade-specific life expectancy, i.e., for lack of a better assumption, the survival functions for cohorts born between 1790 and 1850 are assumed to have the same shape as the 1850 survival function.

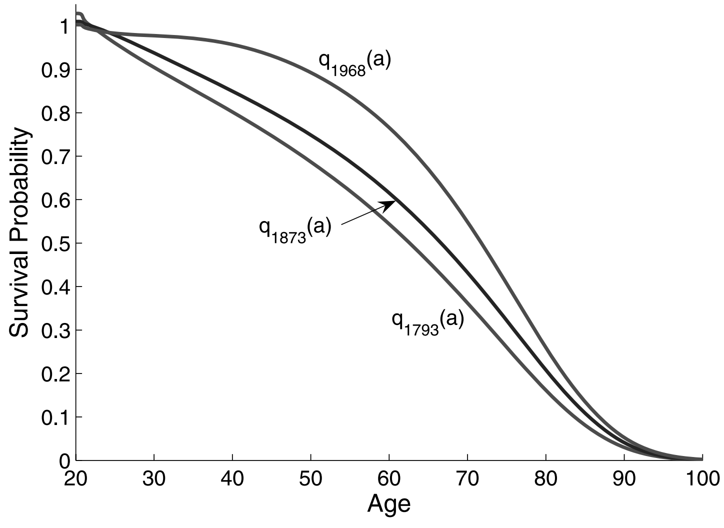

FIGURE 5

SURVIVAL FUNCTIONS OF THE 1973 COHORT, 1848 COHORT, AND 1968 COHORT

the 1968 cohort versus both the 1793 cohort and the 1873 cohort is captured by the rounding out of the survival functions.

Education distributions. The cohort-specific distributions of agents across education groups, $F_\tau(e)$, are calibrated using U.S. Census data on years of school completed by age for males. Since the data is only available every 10 years starting in 1940, the distributions are set as follows. For the 1923–1998 cohorts, each distribution is chosen to match that cohort's corresponding distribution when it is either 30–34 years of age or 35–39 years of age from the data. For the 1888–1918 cohorts, each distribution is set to that cohort's distribution in 1940 in the data. Finally, for the 1793–1883 cohorts, the distributions are determined as follows. First, assume that the fraction of males completing high school grew at a constant rate and the fraction of males completing grammar school fell at a constant rate across the 1793–1928 cohorts. Then compute the trendline using the 1888–1928 data. Finally, the fractions completing only high school and grammar school are found by extending the trendline back to 1793. The cohort-specific distributions of agents across education groups are summarized in Figure 6 .

Market productivity profiles and distributions. The market productivity profile of an agent of type s is assumed to be hump-shaped: It reaches its peak height $\tilde{x}_s$ when he is age $\tilde{a}_s$. From the agent's expected age of survival, $\bar{T}_s$, to the maximum age that can be achieved, T, the agent's productivity declines at the constant rate ρ_s. Thus the agent's profile is given by

$$x_s(a) = \begin{cases} \nu_s(a - \tilde{a}_s)^2 + \tilde{x}_s, & 0 \le a \le \bar{T}_s, \\ \Omega_s e^{-\rho_s a}, & \bar{T}_s < a \le T. \end{cases} \tag{12}$$

Note that an agent's profile depends on his cohort-specific life expectancy, $\bar{T}_s$. Figure 4 provides the sequence of life expectancies under the baseline calibration.

Determining the market productivity profiles and the distributions of agents of each education type across initial productivity levels requires setting, for each education group e, the means and standard deviations of the lognormal distributions over initial productivity levels, μ_e and σ_e, and, for each type s, the profile parameters $\tilde{a}_s$, $\tilde{x}_s$, ν_s, Ω_s, and ρ_s. These parameters are determined

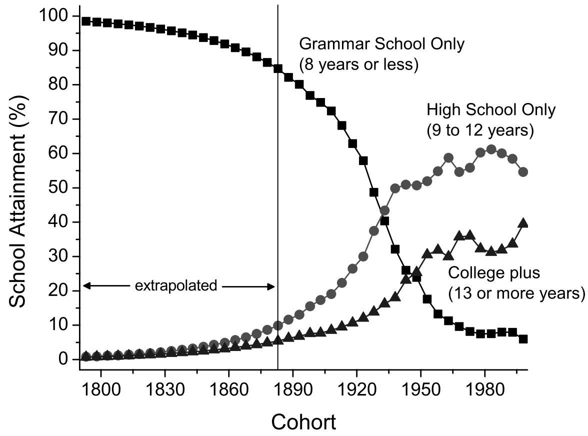

FIGURE 6

DISTRIBUTIONS OF AGENTS ACROSS EDUCATION GROUPS BY COHORT UNDER BASELINE CALIBRATION

TABLE 3

EARNINGS STATISTICS FROM U.S. CENSUS DATA TARGETED TO CALIBRATE BASELINE MARKET PRODUCTIVITY PROFILES AND INITIAL PRODUCTIVITY DISTRIBUTIONS

		Education Group (e)		
		Grammar School (G)	High School (H)	College (C)
$\bar{E}_e$	Relative mean earnings of 45–49-year-olds	1.00	1.31	1.99
c_e	Coefficient of variation of earnings	0.65	0.67	0.74
f_e	Fraction of way through life of peak earnings	0.54	0.58	0.65
p_e	Peak earnings relative to initial	1.76	1.79	2.54

simultaneously such that the model matches a set of statistics computed from data. The data used are cross-sectional data on the labor earnings of year-round, full-time male workers in 1975 by years of education from the U.S. Census.[16] Of course, the profiles computed from the data are a proxy for productivity conditional on working. In order to mitigate the effect of this discrepancy, the target statistics are based on the earnings of men aged 25–55. Profiles are not constructed for each individual cohort because data on earnings by age and education level are unavailable for earlier years.

The statistics computed from the data and matched by the model are (i) mean earnings of 45–49-year-old males with 9–12 years of education (high school) relative to those with 8 or less (grammar school), denoted $\bar{E}_H$; (ii) mean earnings of 45–49-year-old males with 13 or more years of education (college) relative to those with 8 or less, denoted $\bar{E}_C$; (iii) the average coefficient of variation of earnings over the life cycle by education group, denoted c_e; (iv) the fraction of the way through expected adult life at which earnings peak by education group, denoted f_e; and (v) the ratio of peak earnings to initial earnings by education group, denoted p_e. Table 3 shows the target values. Notice that, on average, the earnings of individuals with more education peak later in their life and reach a higher level relative to their initial earnings.

Given these statistics from the data, the parameters are determined as follows. Mean initial log productivity of the grammar school group is normalized to one. The means for the high

[16] The specific source is U.S. Bureau of the Census, Current Population Reports, Series P-60, No. 105 "Money Income in 1975 of Families and Persons in the United States," U.S. Government Printing Office, Washington, DC, 1977. Fuster et al. (2007) also use cross-sectional earnings data for full-time workers by education group as proxies for education-specific productivity profiles. The average profile is similar to that estimated by Hansen (1993) and commonly used in the literature.

school and college groups and the standard deviations of initial log productivity are chosen such that the mean (across agents and generations) productivity of agents at age 47 in the high school (college) group relative to the grammar school group is equal to $\bar{E}_H$ ($\bar{E}_C$), and for each education group e, the average (over the life cycle and across generations) coefficient of variation of productivity is equal to c_e. The fraction of the way through adult life at which productivity peaks is assumed to be constant across agents of the same education level regardless of the generation to which they belong. Thus, for each education group e, $\tilde{a}_s$ is given by

$$\tilde{a}_s = f_e \bar{T}_s.$$

The ratio of peak earnings to initial earnings is assumed to be constant across agents of the same education level. However, since earlier generations work fewer years before their productivity peaks, the ratio is not assumed to be constant across generations. Instead, the p_e's are only used to determine the ratios for the youngest cohort in the economy, the 1968 cohort. For each type s agent born in 1968 with education level e and initial productivity x_0, peak earnings $\tilde{x}_s$, is set such that

$$\frac{\tilde{x}_s}{x_0} = p_e,$$

and ν_s is set such that

$$x_s(0) = x_0.$$

For each agent born before 1968 of type $\hat{s}$, $\nu_{\hat{s}}$ is set such that

$$x'_{\hat{s}}(0) = x'_s(0),$$

where types $\hat{s}$ and s have the same education and initial productivity level and $\tilde{x}_{\hat{s}}$ is set such that

$$x_{\hat{s}}(0) = x_0.$$

In other words, agents with the same education and initial productivity level are assumed to have the same initial slope. This assumption together with the restriction that initial productivity must be given by x_0 pins down ν_s and $\tilde{x}_s$ for all the cohorts born before 1968. Finally for each type s agent, Ω_s and ρ_s are calculated by forcing the productivity profiles to be smooth and continuous at $\bar{T}_s$, i.e., such that

$$\nu_s(\bar{T}_s - \tilde{a}_s)^2 + \tilde{x}_s = \Omega_s e^{-\rho_s \bar{T}_s},$$

and

$$2\nu_s(\bar{T}_s - \tilde{a}_s) = -\rho_s \Omega_s e^{-\rho_s \bar{T}_s}.$$

The left-hand panel of Figure 7 shows the productivity profiles of agents from the 1903 and 1943 cohort with the fifth highest initial productivity level within each education group. Multiplying the productivity profiles by wages and hours gives the agents' earnings profiles, which are shown in the right-hand panel. Note that, due to wage growth, earnings peak later than productivity. Also, note that the profiles of agents in higher education groups are steeper.

Additional parameters. Five additional parameters that were determined directly from the data are summarized in Table 4. The rate of time preference, θ, is set such that the average value of $\theta + \eta$ equals 0.02, or, in other words, the average annual discount factor is 0.98. The annual

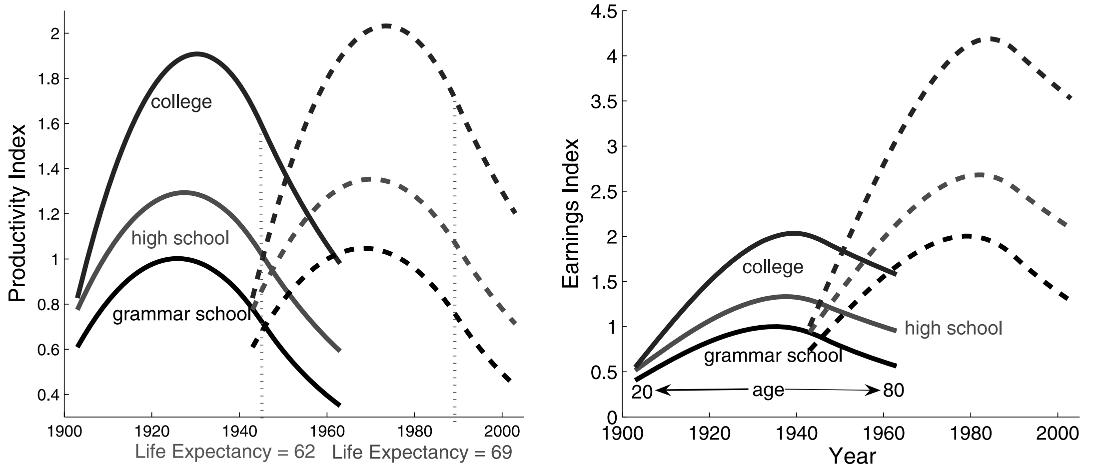

FIGURE 7

SIXTY-YEAR PRODUCTIVITY AND EARNINGS PROFILES OF AGENTS FROM THE 1903 AND 1943 COHORTS WITH THE FIFTH HIGHEST INITIAL PRODUCTIVITY LEVELS WITHIN THEIR EDUCATION GROUP. PEAK PRODUCTIVITY AND PEAK EARNINGS OF THE 1903 GRAMMAR SCHOOL AGENT ARE NORMALIZED TO 1

TABLE 4
PARAMETER VALUES UNDER BASELINE CALIBRATION DETERMINED DIRECTLY FROM DATA

Set		
θ	Rate of time preference	0.007
κ	Growth rate of wages	0.015
γ	Rate of price decline	0.006
r	Interest rate	0.041
$\bar{h}$	Fraction of time spent working	0.46

growth rate of wages of 1.5% is for the period 1830–2000. It was determined using the real wage index of Williamson (1995) for the period 1830–1988 and BLS data for the period 1988–2000. Similarly, the rate at which the price of leisure goods falls is estimated from the leisure price series presented in Figure 3. The 4.1% annual interest rate is an after-tax rate and is taken from McGrattan and Prescott (2000). The fraction of time spent working is set to 46%. This is the average time spent working of males in the United States over the period 1830–2000.[17]

4.2.2. *Estimation*. The rest of the parameters are chosen such that the model matches the data along eight moments. The first six moments are the retirement rates of the six cohorts alive in the year 2000.[18] The empirical retirement rates were computed using data from the 2000 HRS and are reported in Table 5. The year 2000 retirement rates are similar to those shown in Section 2.1 found using IPUMS data. The seventh moment is the median drop in market consumption. The target is taken from Hurd and Rohwedder (2008), who find a median drop for individuals of 5.9% of preretirement expenditure. Their estimate of the consumption drop is the first one based on observations of total expenditure of individuals before and after retirement. The eighth moment is leisure goods' share of total expenditure in 2000. The empirical value is set to 11.8%, which is the share of total expenditure allocated to leisure goods according

[17] Data on weekly hours worked by U.S. males is from Whaples (1990) and the *Statistical Abstracts of the United States*.

[18] The retirement rate is defined in Section 2. The retirement rates for the six cohorts were computed following the same procedure used to construct the retirement rates series in Figure 2.

TABLE 5
MOMENTS: MODEL AND DATA

Cohort	Year When Age 20	Age in 2000	% Retired in 2000	
			Data	Model
1	1941–45	75–79	82.9	83.5
2	1945–50	70–74	75.6	76.6
3	1951–55	65–69	63.9	65.4
4	1956–60	60–64	38.4	43.9
5	1961–65	55–59	16.7	12.7
6	1966–70	50–54	7.5	0
Median consumption drop			5.9	5.9
Leisure share			11.8	15.9

to the BLS and Lebergott (1996) plus 30% of transportation's expenditure share. Both the consumption drop at retirement and leisure goods' share of total expenditure are discussed in more detail in Section 2.2.

The minimization is done as follows. Assign the numbers 1–6 to the six cohorts who are between the ages of 52 and 77 in the year 2000, respectively. Then define the following vector of unknown parameters:

$$\delta = (\hat{c}, \alpha, \zeta, \chi, \sigma, \mu_z, \sigma_z, p_{g,1793}).$$

Given δ, the model's prediction for the labor force participation rate of cohort i is denoted by $P_i(\delta)$, the model's prediction for the median drop in market consumption is denoted by $D(\delta)$, and the model's prediction for leisure goods' share of total expenditure is denoted by $L(\delta)$. The corresponding values in the data are denoted by d, p_i, and l, respectively. The exercise now consists of two steps. First, δ is chosen to minimize the sum of the deviations between the model's output and the empirical moments. Formally,

$$\hat{\delta} = \arg\min_{\delta} \left\{ (d - D(\delta))^2 + (l - L(\delta))^2 + \sum_{i=1}^{6} (p_i - P_i(\delta))^2 \right\}.$$

Second, the model's predictions, $D(\hat{\delta})$, $L(\hat{\delta})$, and $P_i(\hat{\delta})$, for $i = 1, \ldots, 6$, are computed using $\hat{\delta}$.

The results of the minimization are shown in Table 5. Even though there are eight moments and eight parameters, the model is unable to match the moments perfectly. Still, the model is able to generate the dispersion in retirement rates across age groups observed in the data.

Notice that the model has more difficulty matching the retirement rates of the younger age groups than the older ones. For example, in the model, 43.9% of 60–64-year-olds are retired in 2000 compared with 38.4% in the data. The overestimation of the retirement rates of 60–64-year-olds by the model may occur because the model does not account for the impact of Social Security on retirement. As documented in Gustman and Steinmeier (2005), the hazard rate for retirement spikes at ages 62 and 65 in the United States. These are the ages at which U.S. workers first become eligible for Social Security benefits and can receive benefits without an early retirement reduction penalty, respectively. The hazard rates suggest that individuals may delay or advance their retirement in order to retire at these particular ages. If there are additional benefits to retiring at ages 62 and 65 not taken into account in the model, the individuals that would most likely adjust their retirement are those with the smallest cost, i.e., those whose optimal retirement age without Social Security is close to either age 62 or age 65. Although the impact of delaying or advancing retirement to age 62 on the model's predictions should be small, since individuals aged 60–64 are lumped together, the impact of the spike at 65 will

TABLE 6
PARAMETER VALUES USED IN BASELINE MODEL THAT WERE CHOSEN TO MATCH THE MODEL TO MOMENTS BASED ON DATA FROM THE 2000 HRS

Calibrated in Minimization		
$\hat{c}$	Subsistence consumption level	0.042*
α	Market consumption's share of total consumption	0.33
ξ	Weight on leisure goods in leisure production function	0.073
χ	Determines elasticity of substitution between leisure goods and leisure time	−1.72
σ	Determines intertemporal elasticity of substitution	1.32
μ_z	Mean of log leisure productivity distribution	0.77
σ_z	Std. dev. of log leisure productivity distribution	1.89
$p_{g,1793}$	1793 price of leisure goods	8.75

*Relative to mean annual income of 30–34-year-olds in 1963.

not be. If some HRS respondents who, in a world without Social Security, would have chosen to retire at ages close to 65 delayed their retirement to avoid the early retirement reduction penalty, then retirement rates from the model would overestimate those observed in the data for the 60–64-year-olds.

The model underestimates the retirement rates of the 55–59-year-olds and the 50–54-year-olds. In particular, in the model, no 50–54-year-olds are retired, whereas 7.5% of them are retired in the data. One possible reason for the underestimation is that the model abstracts from intragenerational heterogeneity in life expectancies. In the data, life expectancy is positively correlated with income. For example, De Nardi et al. (2006) find a 3-year differential in the life expectancies of men in the 20th income percentile compared to men in the 80th at age 70. If this variation in life expectancies was incorporated into the model, it would lower the retirement age of low productivity types. The model's difficulty in matching the retirement rates of the younger age groups may also be due to the fact that retirement before age 60 is more likely to be due to medical conditions that make continuing to work difficult. In addition, starting in 1950, workers with medical conditions who retired were eligible for Social Security Disability Insurance (SSDI) benefits. Poor health combined with SSDI may have provided additional incentives to retire early that are not captured in the model.

The model is able to generate the median consumption drop at retirement observed in the data. However, leisure goods' share of expenditure in the model (15.9%) is larger than the share targeted in the data (11.8%). This may be due to the functional form chosen for utility. The assumption that momentary utility is a Cobb–Douglas of market consumption net of subsistence and leisure forces market consumption net of subsistence and nonmarket time to have the same degree of complimentarily as market consumption net of subsistence and leisure good consumption. Relaxing this assumption could improve the model's ability to match this target, but at a cost of additional complexity. On the other hand, 15.8 is not implausible when one considers that there are many leisure expenditures that have not been included in the set of leisure goods given by the BLS. One such example is vacation spending, which was 4.2% of total expenditure by individuals aged 65–74 in 1993.

The values of the parameters that were chosen through the minimization procedure are given in Table 6. The subsistence consumption level is equivalent to approximately 5 dollars a day in year 2000. Note that the values for α and σ imply that the intertemporal elasticity of substitution for market consumption net of subsistence is 0.81. This value is well within the range suggested

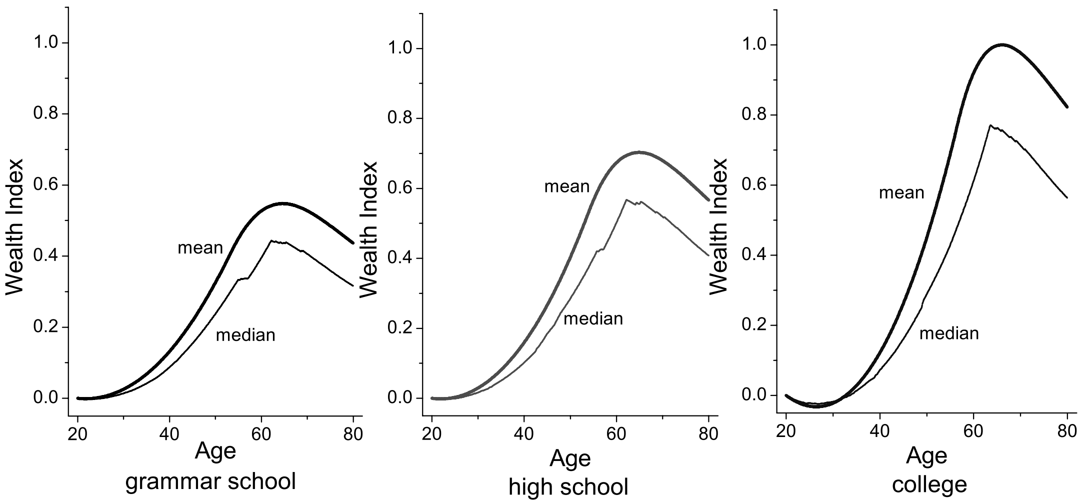

FIGURE 8

LIFE CYCLE PROFILES OF MEAN AND MEDIAN WEALTH BY EDUCATION GROUP IN THE MODEL

in the literature.[19] The baseline calibration is also consistent with the finding of Weagley and Huh (2004) discussed in Section 2.2.1, as the increase in leisure time that occurs at the moment of retirement generates a contemporaneous jump in expenditure on leisure goods.

Figure 8 shows the profiles of mean and median wealth over the life cycle for each education group from the 1943 cohort. Consistent with U.S. data, the profiles are hump-shaped and peak at the common retirement age of 65. Notice that individuals in higher education groups are wealthier on average. This is consistent with Survey of Consumer Finance (SCF) and Panel Study of Income Dynamics data as documented by Cagetti (2003). In addition, the mean profile is above the median, illustrating that wealth is skewed across individuals in the same cohort. Aggregating across education groups, the ratio of the mean to median profile at the point when the profiles peak is 1.4. Fernández-Villaverde and Krueger (2005) find, using cross-sectional data from the SCF, that this ratio is about 4 for U.S household in 1995. Thus, there is significantly less wealth inequality in the model than in the data. However, this is not surprising given that the model abstracts from many mechanisms that have been shown to be important drivers of wealth inequality in the data such as the Social Security program, means-tested social insurance programs, and income, medical expense, and survival risk.

4.3. *Evolution of Retirement.* The model's prediction for the trend in retirement was obtained by running the calibrated model over the time period from 1793 to 2000. The results are presented in Figure 9. The model predicts that the retirement rates of the age groups above 60 increased steadily over the 150 years. In the data, the retirement rate of men aged 75–79 goes from approximately 22% in 1850 to 85% in 2000, an increase of 62 percentage points. In the model, the rate is 23% in 1850 and 84% in 2000, an increase of 60 percentage points. Thus the model captures 96% of the increase in retirement of this cohort. By making a similar calculation, the model explains 99% of the total rise in retirement of the 70–74-year-olds, and 87% of the total rise of the 65–69-year-olds.

Although the model is able to generate a large share of the overall increase in retirement of men 65 and over for the 1850–2000 period, it underestimates the rate in the later years and overestimates the rate in the early years. In addition, the model is unable to generate the

[19] For example Attanasio and Weber (1993) find that the IES of consumption should be in the range from 0.3 to 0.8 based on microdata whereas values as high as 1 are common in real business cycle literature. See Guvenen (2006) for an interesting discussion.

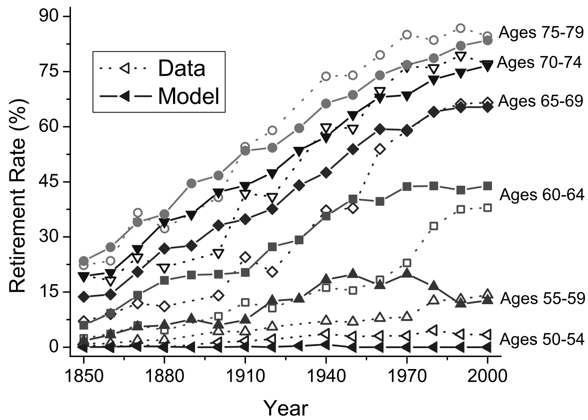

FIGURE 9

MODEL'S PREDICTION FOR THE TREND IN RETIREMENT BY AGE GROUP VERSUS THE TRENDS IN THE DATA

leveling out of retirement occurring from 1980 to 2000. Note that, in the model, the real wage and relative price of leisure goods change at constant rates. Thus the model is designed to assess the impact of the long-run trends in wage rates and leisure good prices. The impact of short-run fluctuations of these prices is not captured.

Although the model has a difficult time matching the observed retirement rate levels of men aged 55–64, it still predicts a large share of the overall rise in their retirement rates over the 1850–2000 period. For example, the model overestimates the retirement rate levels of the 60–64-year-olds in nearly every period. However the retirement rate increases by 38 percentage points in the model compared to 36 in the data. The overestimation of the levels may be due, as mentioned above, to the fact that the model abstracts from Social Security, which may cause men in this age group to delay retirement until 65. The model also has difficulty capturing the steep rise in this group's retirement rates after 1970. The steep rise may be driven by an increase in private pension plans during this period that is not captured in the model. From their structural estimation, Anderson et al. (1999) find that over the 1960–1980 period increases in pensions and Social Security can account for about a quarter of the total increase in retirement of individuals under age 65.

In the model, changes in life expectancy have a big impact on retirement between ages 55 and 59. The decreasing retirement rates for this age group starting in 1950 are driven by a large increase in the group's life expectancy. With retirement rates decreasing, in order to match the retirement rate for this age group in 2000, the model must overestimate the retirement rate for the preceding years. Despite the model's inability to match the trend, it does predict that the retirement rate rises from approximately 2% in 1850 to 13% in 2000 compared with 1% and 14% in the data, capturing 96% of the overall rise.

The model predicts that the retirement rates for 50–54-year-olds are zero in all periods, underestimating the rates in the data. As mentioned above, the model abstracts from negative health shocks and variations in life expectancy, both of which may be important in accounting for retirement before age 55.

4.4. *Counterfactuals.* In order to better understand how the combination of rising real wages, falling relative prices of leisure goods, rising education levels, and increases in life expectancy drive the trends of rising retirement of the six cohorts under the baseline calibration, a series of counterfactual experiments is conducted. These experiments consist of "shutting-down" one or two of the driving forces at a time, otherwise maintaining the baseline calibration, and rerunning the 150 year transition.

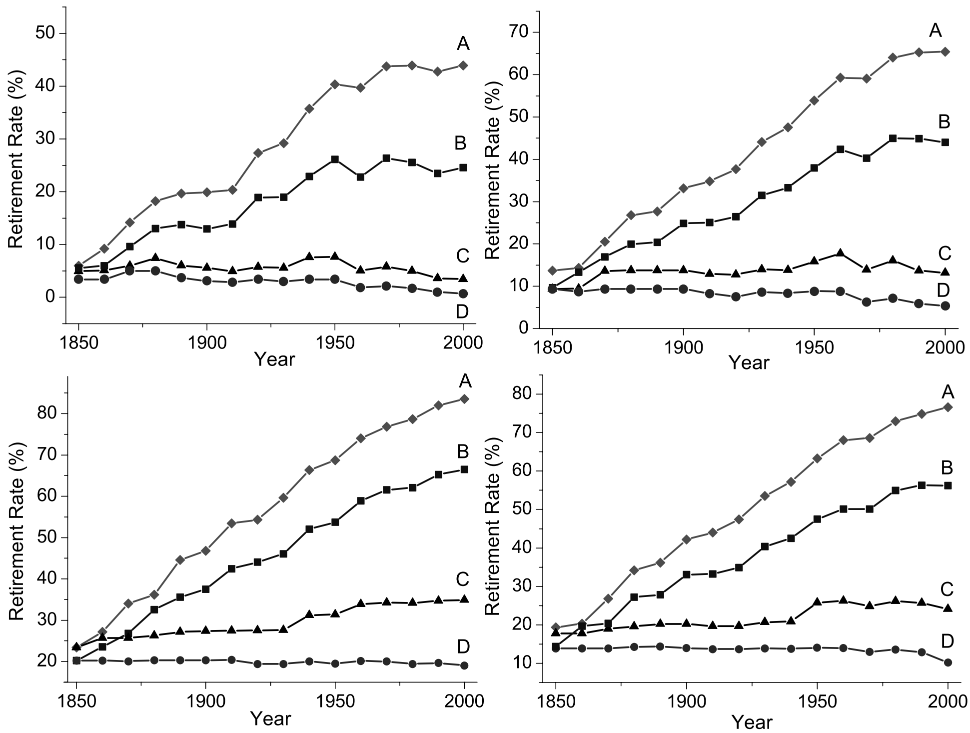

FIGURE 10

THE MODEL'S PREDICTION FOR THE RETIREMENT RATES OF MEN AGED 60–64 (UPPER LEFT), AGED 65–69 (UPPER RIGHT), AGED 70–74 (LOWER LEFT), AND AGED 75–79 (LOWER RIGHT) UNDER THE BASELINE CALIBRATION (LINES A), WHEN BOTH THE PRICE OF LEISURE GOODS AND AGENTS' INITIAL WAGES REMAIN AT THEIR 1793 VALUE (LINES D), WHEN ONLY THE PRICE OF LEISURE GOODS REMAINS AT ITS 1793 VALUE (LINES B), AND WHEN ONLY THE INITIAL WAGE REMAINS AT ITS 1793 VALUE (LINES C)

The growth in wages has two effects on retirement. The first effect comes from the fact that wages are increasing across generations or, in other words, the initial wage of cohorts born at later dates in time is higher than that of those born earlier. Since χ is negative under the baseline calibration, by Proposition 2, the increase in the level of wages over time will have a positive effect on retirement. The second effect comes from the fact that the wage rate is growing throughout a cohort's lifetime, impacting the shapes of the earnings profiles of members of the cohort. Changes in the shape of an agent's earnings profile change the amount of labor earnings the agent loses by retiring at any particular moment, or the marginal cost of retiring, thus altering his retirement date. In order to isolate the two effects, in the first set of counterfactual experiments, only agents' initial wage level will be kept fixed at its 1793 value. Wages will still increase over the agents' lifetimes so that the shape of the agents' earnings profiles will remain as in the baseline. This way only the impact of the rise in wages across generations is shut down.

The results of the first three counterfactual experiments for the retirement rates of the four older age groups are summarized by the four panels in Figure 10. The lines labeled A show the retirement rates of each of these four groups under the baseline calibration. Lines labeled D show the retirement rates in an economy where both the rise in agents' initial wage and the fall in the price of leisure goods since 1793 are "shut down," or kept at their 1793 value. With both agents' initial wage and the price of leisure goods fixed at their 1793 value the only forces impacting retirement over time are the changes in the agents' survival profiles, income profiles due to changes in life expectancy, and education levels. Notice that for all four age groups, retirement rates decrease instead of increase over time in this economy. The direct effect of increasing an agent's life expectancy is an increase in his retirement age. In order to pay for the expected extra years of consumption, it is optimal for the agent to increase his lifetime income by retiring later. But changing an agent's life expectancy also changes the age of at which he reaches peak market productivity and the maximum height of his productivity profile. This has two effects on the agent's retirement decision. On one hand, since his market productivity peaks at a later age, it rises to a higher level, making him wealthier at a younger age and, given that χ is negative, more likely to retire earlier. On the other hand, the higher peak means the agent's productivity at the moment he used to retire is now higher and therefore the marginal cost of retiring at this moment, in terms of forgone earnings, has gone up. The net effect of the increase in survival rates is a reduction in retirement. The increase in the fraction of individuals in the higher education groups generates a slight increase in retirement. Switching an agent into a higher education group, holding his relative position within the group constant, increases his initial productivity level. Again, with χ negative, this will increase the benefit from retiring earlier. However, the switch also increases the age at which his productivity peaks and the level it obtains relative to his initial productivity. This will increase the cost of retiring. The net effect of the rise in education is a slight increase in retirement. Finally, the total effect of the increases in life expectancy and education over the period is to decrease retirement rates over time.

The lines labeled B show the retirement rates in an economy where only the falling price of leisure goods is shut down. When the price of leisure goods stays at its 1793 value, the fraction of each cohort that is retired is significantly lower than in the baseline. The agents can no longer afford to purchase as many leisure goods as before, and, since leisure goods are compliments with leisure time, this reduces the marginal benefit of retiring. The lines labeled C show the retirement rates for the case where only agents' initial wage is kept fixed at its 1793 value. Since χ negative generates a dominant income effect, when real wages do not increase, the percentage of individuals retired in each period is lower.

Without the fall in the price of leisure goods and the rise in wages, the model predicts that the percentage of 60–64-year-olds that would be retired in 2000 would be 1% compared with 44% in the baseline, a difference of 43 percentage points. How much of that difference is due to the rise in real wages and how much is due to the fall in the price of leisure goods? If only wages had risen and the price of leisure goods had remained unchanged, then the fraction retired in 2000 would have been 25%. Hence the rise in wages alone accounts for 55% of the difference. Similarly, since the retirement rate of 60–64-year-olds in 2000 would have been 3% if only the price of leisure goods had fallen, it alone accounts for 6% of the difference. The rest of the difference, approximately 39%, is due to the interaction of rising real wages and falling prices of leisure goods. For 65–69-year-olds, wages alone can generate 64% of the increase in retirement in 2000 from that observed in an economy with only changes in life expectancy and education and that under the baseline. The fall in the price of leisure goods can generate 13% of the rise alone, leaving approximately 23% due to the interaction of the two forces. For 70–74-year-olds, the contribution of rising wages alone is 69% and of falling leisure good prices alone is 21% with 10% due to the interaction of the two. For 75–79-year-olds rising wages alone contribute 74% and falling leisure prices alone contribute 24%.

5. RELATION TO LITERATURE

The most commonly mentioned hypothesis of the increase in retirement in the United States is that it was due to the development and growth of Social Security programs and pension plans. However, evidence based on empirical studies is mixed, and, whereas many studies find a significant impact of such programs on retirement behavior, they conclude that other factors driving retirement must exist. For example, Lumsdaine et al. (1994) find that whereas changes in pension plans have a significant effect on retirement, Social Security has only a modest effect. Krueger and Pischke (1992) use data from the Current Population Survey to estimate the effect of Social Security wealth on the labor supply of older U.S. males. They find that growth in Social Security benefits can explain less than one sixth of the decline in male labor force participation rates during the 1970s. Anderson et al. (1999) simulate a structural model of retirement and find that increases in pensions and Social Security can account for about a quarter of the total trend toward earlier retirement observed from 1960 to 1980 but had no effect on retirement by those above age 65. Finally, Lee (1998) finds, looking at an earlier time period, that the development of two public welfare programs (Union Army Pensions and Old Age Assistance) cannot explain much of the rise in retirement prior to 1940 and are at most of secondary importance in driving the rise.

Costa (1998) is the first to argue that rising wages along with a fall in the cost of leisure goods and activities and an increase in their variety has been an important driver of the rise in retirement. Her comprehensive study contains empirical evidence supporting this hypothesis. However, the notion that the price of leisure goods has a significant impact on the demand for leisure time was first pointed out by Owen (1971). Owen argues that a significant amount, about 25%, of the decline in weekly hours of U.S. males during the period 1901–1961 is due to the falling relative price of recreation goods. He argues that the other 75% of the decline is due to rises in the real hourly wage. More recently, Vandenbroucke (2009) calibrates a model in which agents produce and derive utility from leisure. Vandenbroucke finds that the decline in the price of leisure goods during the first half of the 20th century can explain a significant, albeit smaller, part of the decline in weekly hours per worker. Also in a recent empirical study, González Chapela (2007) finds that males adjust hours along the intensive margin in response to changing prices of leisure goods. Specifically, he finds that the elasticity of intertemporal substitution of market time with respect to the price of leisure goods is approximately 0.16 and statistically different from zero.

This work is the first attempt to account for the long-run rise in retirement using a quantitative macroeconomic approach. Consistent with Costa (1998), rising wages and falling prices of leisure goods are found to be important drivers of the increase in male retirement since 1850. The finding that changes in leisure good prices are a significant determinant of labor force participation along the extensive margin is also consistent with Vandenbroucke's and Gonzalez-Chapela's findings that changing leisure good prices impact labor supply decisions on the intensive margin.

6. CONCLUSION

In order to assess the ability of rising real wages and declining prices of leisure goods to drive a decrease in labor force participation rates of elderly U.S. males, a model economy in which agents choose the moment of their retirement is developed. Agents in the economy produce leisure by combining leisure time with leisure goods. Under the baseline calibration, complimentarity between leisure time and leisure goods along with a dominant income effect result in the observed increase in real wages and fall in the price of leisure goods driving a long-run trend of rising retirement.

The baseline calibration is obtained using data from the HRS by minimizing the distance between the model and data along eight moments: the retirement rates of six age groups and the median drop in consumption and leisure goods' expenditure share in 2000. The calibrated model is then used to recreate the evolution of the retirement rates of the six groups over the period 1850–2000 by plugging in the growth rates of wages and leisure good prices as well as cohort-specific survival functions, life expectancies, and distributions across education groups from the data. The model is able to explain more than 87% of the rise in retirement for men above 65 since 1850. The model also suggests that these factors had a large impact on the increase in retirement of men between 55 and 64 years of age. A series of counterfactual experiments reveals that the rise in real wages is the dominant factor driving the rise in retirement in the model economy. However, the fall in the price of leisure goods is a significant force as well.

An important direction of future research is to explore the impact that incorporating additional factors into the model might have. For example, the model abstracts from Social Security, pensions, and social insurance programs such as Medicare and Medicaid. These programs may have also played a role in the rise in retirement, and it would be interesting to access how adding them to the model would change the quantitative results. Also, in the current framework it is assumed that individuals cannot adjust labor supply on the intensive margin nor invest in their human capital. However, these decisions may also be affected by changes in relative prices. Thus, adding them to the baseline model could have important consequences for the model's prediction of retirement rates.

APPENDIX

PROOF OF PROPOSITIONS 1 AND 2. First note that when $\sigma = 1$, utility is separable between market consumption and leisure. Thus, there is no jump in market consumption at the moment of retirement. Totally differentiating the first-order conditions, Equations (5), (6), and (7), with respect to $p_{g,\tau}$ and manipulating yields

$$\frac{dg(a)}{dp_{g,\tau}} = -\Omega\Lambda(a)\frac{g(a)}{p_{g,\tau}}, \tag{A.1}$$

and

$$\frac{dc(a)}{dp_{g,\tau}} = (1-\Omega)\frac{c(a)-\bar{c}}{p_{g,\tau}}, \tag{A.2}$$

where

$$\Omega = \frac{x(A)\bar{h}w(A+\tau)}{x(A)\bar{h}w(A+\tau) - p_g(A+\tau)(\overline{g}_A - \underline{g}_A)},$$

and

$$\Lambda(a) = \frac{n(a)^{\chi}}{n(a)^{\chi} - \chi(1-\zeta)\,l(a)^{\chi}}.$$

Totally differentiating the budget constraint, Equation (4), with respect to $p_{g,\tau}$ and rearranging gives

$$\begin{aligned}\frac{dA}{dp_{g,\tau}} = [e^{-rA}q(A)\Gamma p_{g,\tau}]^{-1} \Bigg[\int_0^A & e^{-ra}q(a)\,p_g(a+\tau)g(a)\,da \\ &+ \int_A^T e^{-ra}q(a)\,p_g(a+\tau)g(a)\,da \\ &+ p_{g,\tau}\int_0^T e^{-ra}q(a)\frac{dc(a)}{dp_{g,\tau}}\,da \\ &+ p_{g,\tau}\int_0^A e^{-ra}q(a)\,p_g(a+\tau)\frac{dg(a)}{dp_{g,\tau}}\,da \\ &+ p_{g,\tau}\int_A^T e^{-ra}q(a)\,p_g(a+\tau)\frac{dg(a)}{dp_{g,\tau}}\,da\Bigg],\end{aligned} \tag{A.3}$$

where

$$\Gamma = x(A)\bar{h}w(A+\tau) - p_g(A+\tau)(\overline{g}_A - \underline{g}_A).$$

Similarly, totally differentiating the first-order conditions with respect to w_τ yields

$$\frac{dg(a)}{dw_\tau} = \Omega\Lambda(a)\frac{g(a)}{w_\tau}, \tag{A.4}$$

and

$$\frac{dc(a)}{dw_\tau} = \Omega\frac{c(a)-\bar{c}}{w_\tau}, \tag{A.5}$$

where Ω and $\Lambda(a)$ are as above, and totally differentiating the budget constraint with respect to w_τ gives

(A.6)
$$\frac{dA}{dw_\tau} = -\left[e^{-rA}q(A)\Gamma w_\tau\right]^{-1}\Bigg[\int_0^A e^{-ra}q(a)\bar{h}x(a)w(a+\tau)\,da - w_\tau\int_0^T e^{-ra}q(a)\frac{dc(a)}{dw_\tau}\,da$$
$$- w_\tau\int_0^A e^{-ra}q(a)p_g(a+\tau)\frac{dg(a)}{dw_\tau}\,da$$
$$- w_\tau\int_A^T e^{-ra}q(a)p_g(a+\tau)\frac{dg(a)}{dw_\tau}\,da\Bigg],$$

where Γ is as above.

When $\chi = 0$, utility is separable in leisure goods and leisure time. Thus, there is no jump at A in leisure good consumption and $\Omega = 1$. It is now trivial to see that $dg(a)/dp_{g,\tau} = -g(a)/w_\tau$, $dc(a)/dp_{g,\tau} = 0$, $dg(a)/dw_\tau = g(a)/w_\tau$, and $dc(a)/dw_\tau = [c(a) - \hat{c}]/w_\tau$ for all a. Consequently $dA/dp_{g,\tau} = 0$ and

$$\frac{dA}{dw_\tau} = -\frac{\hat{c}\displaystyle\int_0^T e^{-ra}q(a)\,da}{e^{-rA}q(A)\Gamma w_\tau} < 0.$$

Evaluating Equation (6) at the moment before and after retirement and combining yields

$$\frac{\overline{g}_A^{\chi-1}}{\zeta\overline{g}_A^{\chi} + (1-\zeta)[z(1-\bar{h})]^{\chi}} = \frac{\underline{g}_A^{\chi-1}}{\zeta\underline{g}_A^{\chi} + (1-\zeta)z^{\chi}}. \tag{A.7}$$

Notice that the LHS is decreasing in $\overline{g}_A$ and the RHS is decreasing in $\underline{g}_A$. Also notice that, since $z > 0$ and $0 < \bar{h} < 1$, when $\overline{g}_A = \underline{g}_A$ the LHS is greater than the RHS when $\chi > 0$ and less than the RHS when $\chi < 0$. Therefore, for the equality to hold, $\overline{g}_A$ must be greater than $\underline{g}_A$ when χ is positive and less than $\underline{g}_A$ when χ is negative. In addition, when $\chi > 0$ it must be that

$$\zeta\overline{g}_A^{\chi} + (1-\zeta)[z(1-\bar{h})]^{\chi} < \zeta\underline{g}_A^{\chi} + (1-\zeta)z^{\chi},$$

and the opposite must hold for $\chi < 0$. It follows that the LHS of the first-order condition for A,

$$\begin{aligned}&\frac{1-\alpha}{\chi}\left\{\ln\left[\zeta\overline{g}_A^{\chi} + (1-\zeta)z^{\chi}\right] - \ln\left[\zeta\overline{g}_A^{\chi} + (1-\zeta)[z(1-\bar{h})]^{\chi}\right]\right\}\\ &\quad = \lambda e^{(\theta-r)A}[x(A)\bar{h}w(A+\tau) - p_g(A+\tau)(\overline{g}_A - \underline{g}_A)],\end{aligned} \tag{A.8}$$

is always positive. Hence $\Gamma > 0$ for all $\chi < 1$. Now notice that when χ is positive $\Omega > 1$ and $\Lambda(a) > 1$ for all a. Thus, for all a, $dg(a)/dp_{g,\tau} < -g(a)/p_{g,\tau}$, $dc(a)/dp_{g,\tau} < 0$, $dg(a)/dw_\tau > g(a)/w_\tau$, and $dc(a)/dw_\tau > [c(a) - \hat{c}]/w_\tau$. As a result $dA/dp_{g,\tau} < 0$. The sign of dA/dw_τ depends on $\hat{c}$. Notice that if $\hat{c} = 0$, then $dA/dw_\tau > 0$. Similarly, when χ is negative $\Omega < 1$, and $0 < \Lambda(a) < 1$ for all a. Therefore, for all a, $dg(a)/dp_{g,\tau} > -g(a)/w_\tau$, $c(a)/dp_{g,\tau} > 0$, $dg(a)/dw_\tau < g(a)/w_\tau$, and $dc(a)/dw_\tau < [c(a) - \hat{c}]/w_\tau \le c(a)/w_\tau$. It follows that $dA/p_{g,\tau} > 0$ and $dA/dw_\tau < 0$. ■

PROOF OF PROPOSITION 3. Totally differentiating the first-order conditions with respect to z and rearranging yields

$$\frac{dg(a)}{dz}=\frac{\Theta p_g(a+\tau)\Lambda(a)n(a)^{\chi}}{\zeta\Gamma g(a)^{\chi-2}}-\frac{\chi(1-\zeta)z^{\chi-1}g(a)}{n(a)^{\chi}-\chi(1-\zeta)l(a)^{\chi}}, \tag{A.9}$$

and

$$\frac{dc(a)}{dz}=\frac{\Theta p_g(a+\tau)n(a)^{\chi}[c(a)-\hat{c}]}{\Gamma\zeta g(a)^{\chi-1}}, \tag{A.10}$$

where

$$\Theta=\frac{(1-\zeta)(1-\bar{h})^{\chi}z^{\chi-1}}{\overline{n}_A^{\chi}}-\frac{(1-\zeta)z^{\chi-1}}{\underline{n}_A^{\chi}},$$

and $\Lambda(a)$ and Γ are as in the proof of Propositions 1 and 2. Totally differentiating the budget constraint with respect to z and rearranging yields

$$\begin{aligned}\frac{dA}{dz}=\left[e^{-rA}q(A)\Gamma\right]^{-1}\Bigg[&\int_0^T e^{-ra}q(a)\frac{dc(a)}{dz}da+\int_0^A e^{-ra}q(a)p_g(a+\tau)\frac{dg(a)}{dz}da\\&+\int_A^T e^{-ra}q(a)p_g(a+\tau)\frac{dg(a)}{dz}da\Bigg].\end{aligned} \tag{A.11}$$

In the proof of Propositions 1 and 2 it is shown that $\Lambda(a)>0$ for all a and $\Gamma>0$. Notice that if $\chi=0$, then $\overline{n}_A=\underline{n}_A$ and $\Theta=0$. Now it is easy to see that if $\chi=0$, then $dg(a)/dz=0$, $dc(a)/dz=0$, and $dA/dz=0$. Using the first-order conditions, Θ can be rewritten as

$$\Theta=\frac{\zeta z\overline{g}_A^{\chi-1}[\underline{g}_A-\overline{g}_A]}{\overline{n}_A^{\chi}}.$$

In the proof of Proposition 1 and 2 it is shown that when χ is positive $\underline{g}_A<\overline{g}_A$ and hence $\Theta<0$. It follows that, for all a, $dg(a)/dz<0$ and $dc(a)/dz<0$. As a result $dA/dz<0$. It is also shown in the preceding proof that when χ is negative $\underline{g}_A>\overline{g}_A$. Hence $\Theta>0$ and, for all a, $dg(a)/dz>0$, $dc(a)/dz>0$, and therefore $dA/dz>0$. ∎

Numerical algorithm. For each type s the agent's maximization problem is solved as follows. Given initial guesses for $g(0)$, $\underline{g}_A$ and A, market consumption at birth and retirement, $c(0)$ and $\underline{c}_A$, and the multiplier, λ, are computed using Equations (5) and (6). Then $\overline{g}_A$ and $\overline{c}_A$ are found by solving the initial value problem (IVP) characterized by Equation (11) with $g(0)$ given. The IVP's are solved using fifth- and sixth-order Runge–Kutta methods. Next the first-order condition for A, Equation (7), and the first-order condition for $g(0)$, Equation (6), are computed. Finally the budget constraint, Equation (4), is computed using Gaussian quadrature and the Runge–Kutta methods. This procedure is iterated upon according to a variation of Newton's method until the equations converge to within the desired tolerance. At this stage, the second-order conditions are checked. In the case where a maximizer is not found, the corner solution is checked, then a grid search over A is begun to find a region where the second-order condition holds. At each point in the A grid, Equation (6) when age is 0 and the budget constraint are used to solve for $g(0)$ and $\underline{g}_A$ using Newton's method-based algorithm. Once a region that bounds the maximizer is found, the secant method is used to compute A to the desired level of accuracy.

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